

Formula Based Performance Design

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Abstract

Performance models are mostly simple mathematical formulas. The challenge is to model the performance at an appropriate level. In this presentation we introduce several levels of modeling, labeled zeroth order, second order, et cetera. AS illiustration we use the performance of MRI reconstruction.

Theory Block: n Order Formulas

0th order main function
parameters *order of magnitude*
relevant for main function

1st order add overhead
secondary function(s) *estimation*

2nd order interference effects
circumstances *main function, overhead
and/or secondary functions*
more accurate, understanding

CPU Time Formula Zero Order

$$t_{\text{cpu total}} = t_{\text{cpu processing}} + t_{\text{UI}}$$

$$t_{\text{cpu processing}} = n_x * n_y * t_{\text{pixel}}$$

CPU Time Formula First Order

$$t_{\text{cpu total}} = t_{\text{cpu processing}} + t_{\text{UI}}$$

$$+ t_{\text{context switch overhead}}$$

CPU Time Formula Second Order

$$t_{\text{cpu total}} = t_{\text{cpu processing}} + t_{\text{UI}} + t_{\text{context switch overhead}} +$$

$$t_{\text{stall time due to cache efficiency}} + t_{\text{stall time due to context switching}}$$

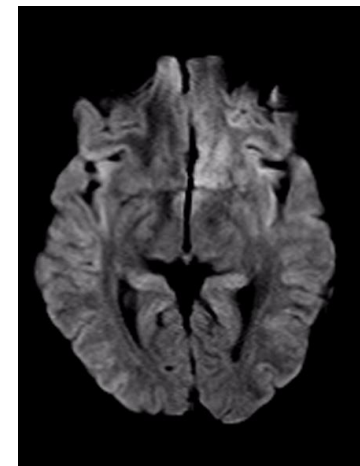
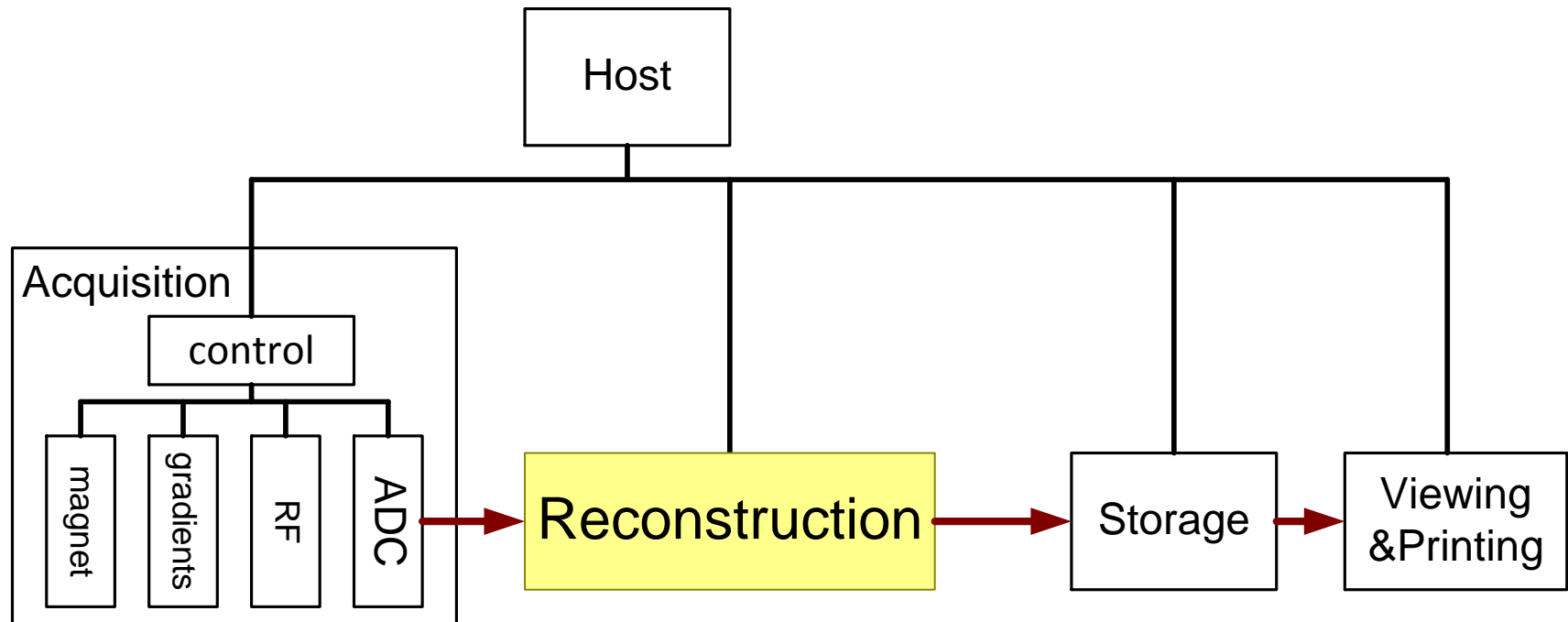
signal processing: high efficiency
control processing: low/medium efficiency

MRI reconstruction

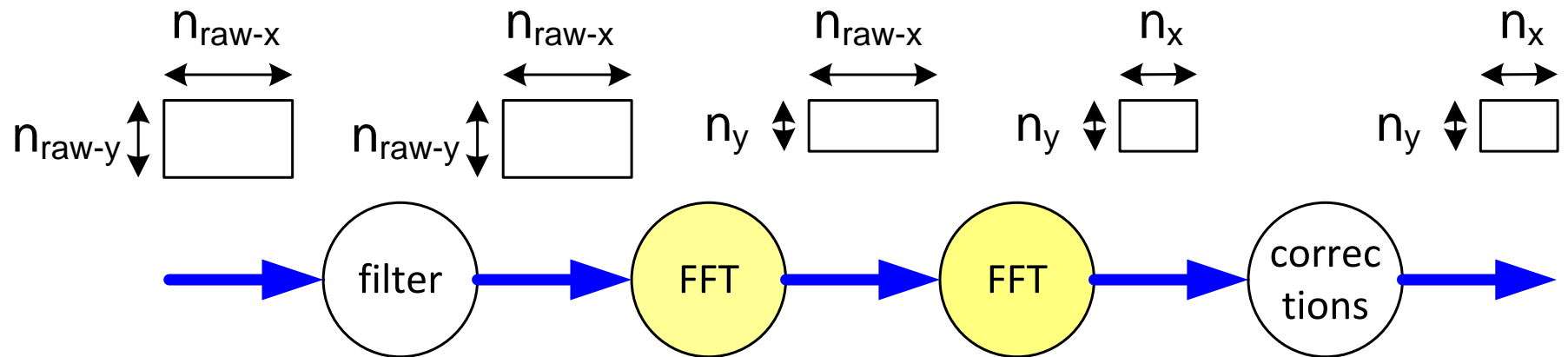
"Test" of performance model on another case

Scope of performance and significance of impact

MR Reconstruction Context



MR Reconstruction Performance Zero Order



$$t_{\text{recon}} = n_{\text{raw-x}} * t_{\text{fft}}(n_{\text{raw-y}}) + n_y * t_{\text{fft}}(n_{\text{raw-x}})$$

$$t_{\text{fft}}(n) = c_{\text{fft}} * n * \log(n)$$

Zero Order Quantitative Example

Typical FFT, 1k points ~ 5 msec
(scales with $2 * n * \log(n)$)

using:

$$n_{\text{raw-x}} = 512$$

$$n_{\text{raw-y}} = 256$$

$$n_x = 256$$

$$n_y = 256$$

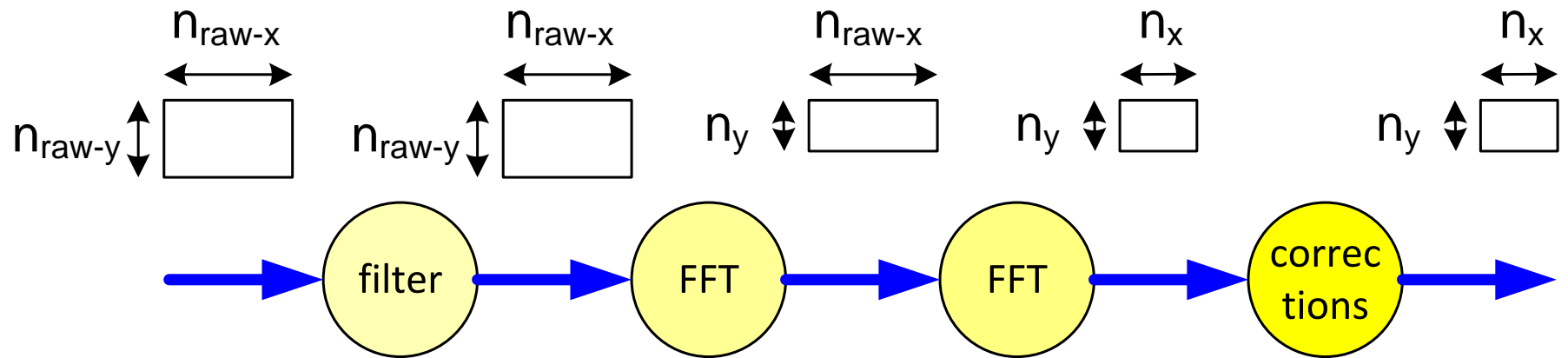
$$t_{\text{recon}} = n_{\text{raw-x}} * t_{\text{fft}}(n_{\text{raw-y}}) +$$

$$n_y * t_{\text{fft}}(n_{\text{raw-x}}) +$$

$$512 * 1.2 + 256 * 2.4$$

$$\sim 1.2 \text{ s}$$

MR Reconstruction Performance First Order



$$t_{\text{recon}} = t_{\text{filter}}(n_{\text{raw-x}}, n_{\text{raw-y}}) + n_{\text{raw-x}} * t_{\text{fft}}(n_{\text{raw-y}}) + n_y * t_{\text{fft}}(n_{\text{raw-x}}) + t_{\text{corrections}}(n_x, n_y)$$

$$t_{\text{fft}}(n) = c_{\text{fft}} * n * \log(n)$$

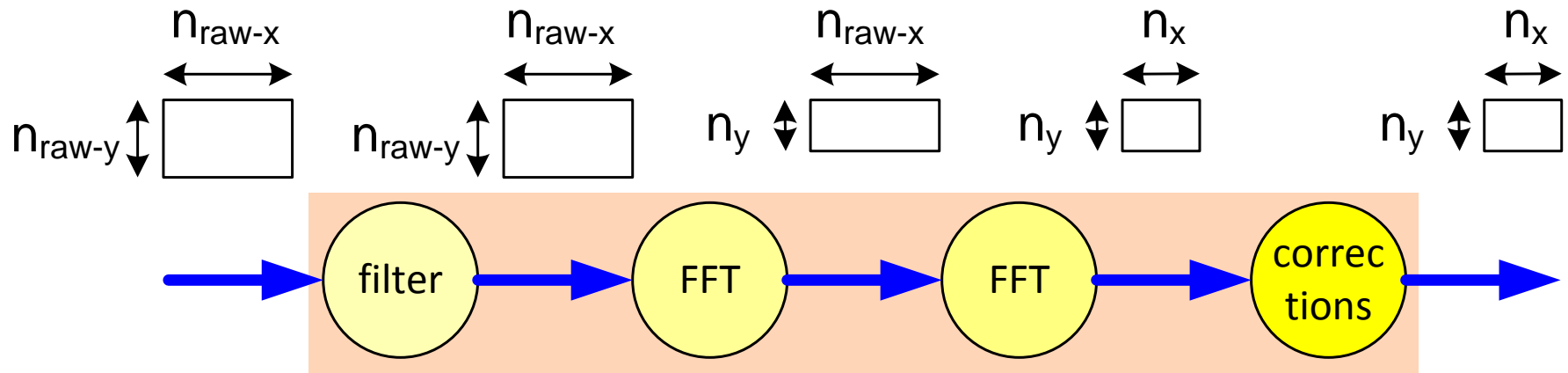
First Order Quantitative Example

Typical FFT, 1k points ~ 5 msec
(scales with $2 * n * \log(n)$)

Filter 1k points ~ 2 msec
(scales linearly with n)

Correction ~ 2 msec
(scales linearly with n)

MR Reconstruction Performance Second Order



$$t_{\text{recon}} = t_{\text{filter}}(n_{\text{raw-x}}, n_{\text{raw-y}}) + n_{\text{raw-x}} * (t_{\text{fft}}(n_{\text{raw-y}}) + t_{\text{col-overhead}}) + n_{\text{y}} * (t_{\text{fft}}(n_{\text{raw-x}}) + t_{\text{row-overhead}}) + t_{\text{corrections}}(n_{\text{x}}, n_{\text{y}}) + t_{\text{control-overhead}}$$

$$t_{\text{fft}}(n) = c_{\text{fft}} * n * \log(n)$$

Second Order Quantitative Example


Typical FFT, 1k points ~ 5 msec
(scales with $2 * n * \log(n)$)

Filter 1k points ~ 2 msec
(scales linearly with n)

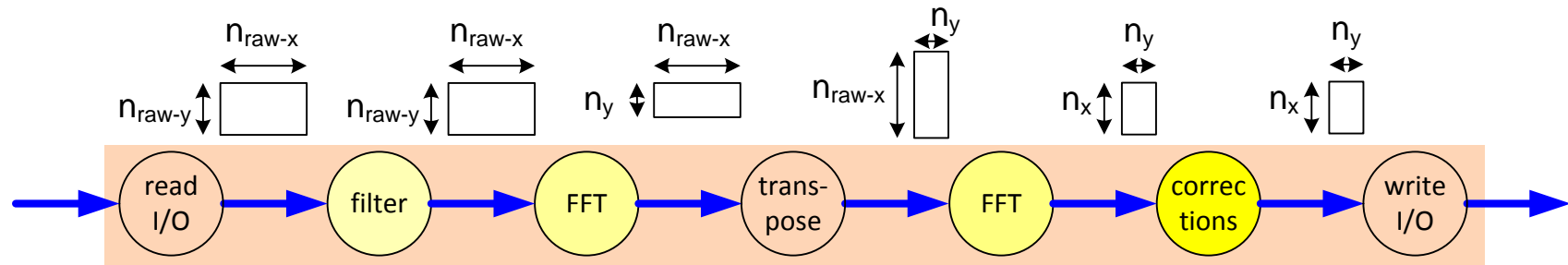
Correction ~ 2 msec
(scales linearly with n)

Control overhead = $n_y * t_{\text{row overhead}}$

10 .. 100 μs



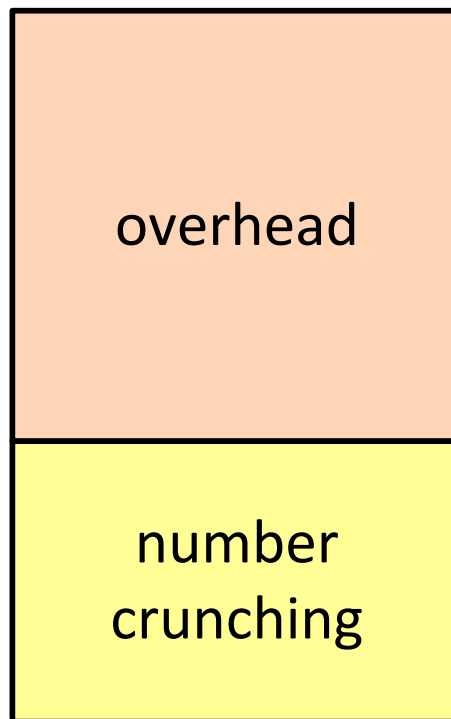
MR Reconstruction Performance Third Order



$$t_{recon} = t_{filter}(n_{raw-x}, n_{raw-y}) + n_{raw-x} * (t_{fft}(n_{raw-y}) + t_{col-overhead}) + n_y * (t_{fft}(n_{raw-x}) + t_{row-overhead}) + t_{corrections}(n_x, n_y) + t_{read\ I/O} + t_{transpose} + t_{write\ I/O} + t_{control-overhead}$$

$t_{fft}(n) = c_{fft} * n * \log(n)$

bookkeeping
transpose
malloc, free
write I/O
read I/O
overhead
correction computations
row overhead
FFT computations
column overhead
FFT computations
overhead
filter computations



focus on overhead reduction

is more important

than faster algorithms

this is not an excuse for sloppy algorithms

MRI reconstruction

System performance may be determined by other than standard facts

E.g. more by overhead I/O rather than optimized core processing

==> Identify & measure what is performance-critical in application

The ASP™ course is partially derived from the EXARCH course developed at *Philips CTT* by *Ton Kostelijk* and *Gerrit Muller*.

Extensions and additional slides have been developed at *ESI* by *Teun Hendriks*, *Roland Mathijssen* and *Gerrit Muller*.