

Physical Models of an Elevator

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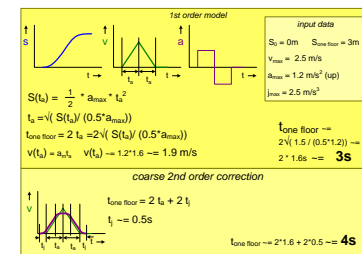
Abstract

An elevator is used as a simple system to model a few physical aspects. We will show simple kinematic models and we will consider energy consumption. These low level models are used to understand (physical) design considerations. Elsewhere we discuss higher level models, such as use cases and throughput, which complement these low level models.

Distribution

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Learning Goals

To understand the need for

- various views, e.g. physical, functional, performance
- mathematical models
- quantified understanding
- assumptions (when input data is unavailable yet) and later validation
- various visualizations, e.g. graphs
- understand and hence model at multiple levels of abstraction
- starting simple and expanding in detail, views, and solutions gradually, based on increased insight

To see the value and the limitations of these conceptual models

To appreciate the complementarity of conceptual models to other forms of modeling, e.g. problem specific models (e.g. structural or thermal analysis), SysML models, or simulations

warning

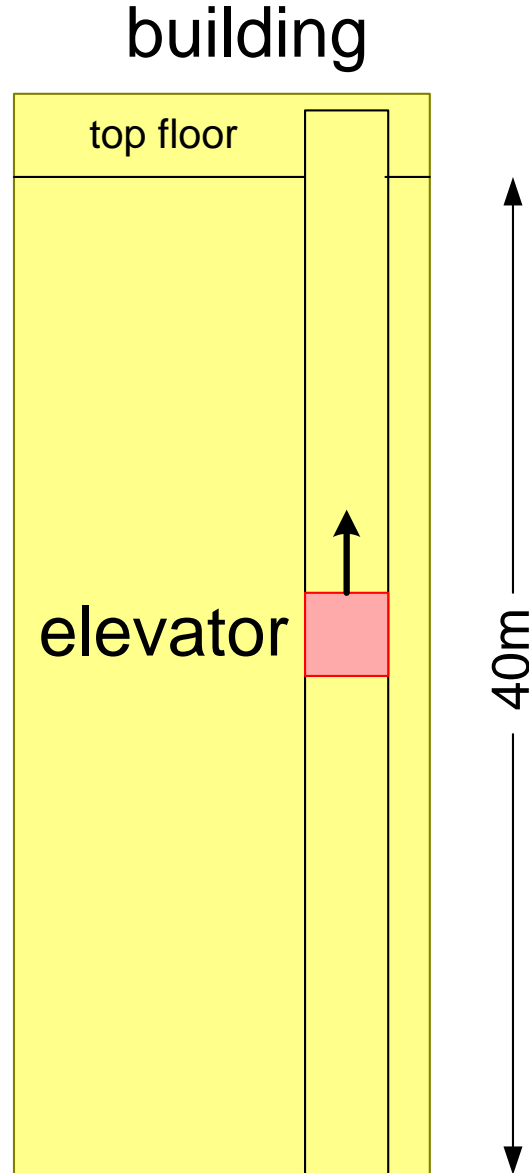
This presentation starts with a trivial problem.

Have patience!

Extensions to the trivial problem are used to illustrate many different modeling aspects.

Feedback on correctness and validity is appreciated

The Elevator in the Building



inhabitants want to reach their destination fast and comfortable

building owner and service operator have economic constraints: space, cost, energy, ...

Elementary Kinematic Formulas

S_t = position at time t

v_t = velocity at time t

a_t = acceleration at time t

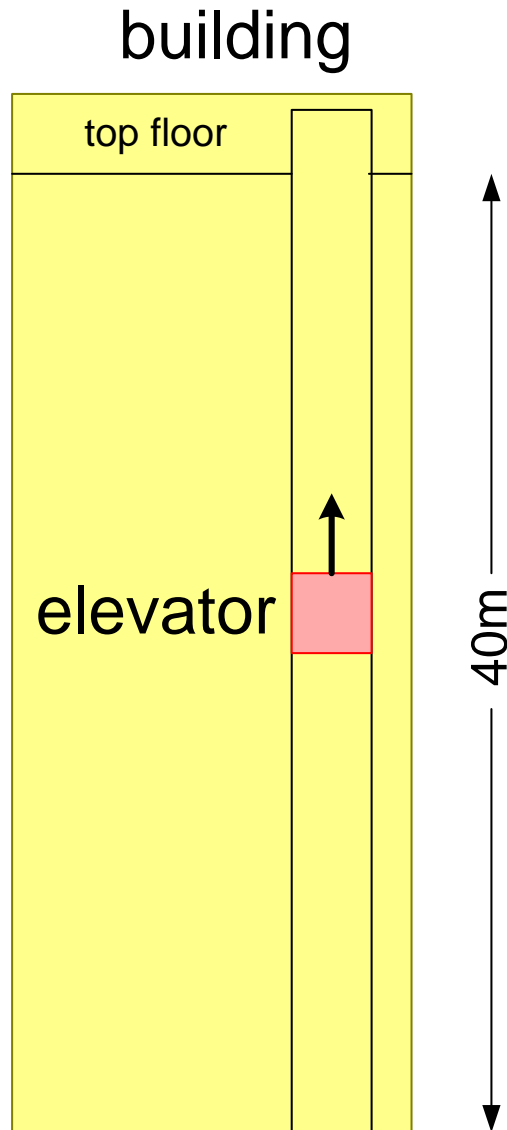
j_t = jerk at time t

$$v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

Initial Expectations



What values do you expect or prefer for these quantities? Why?

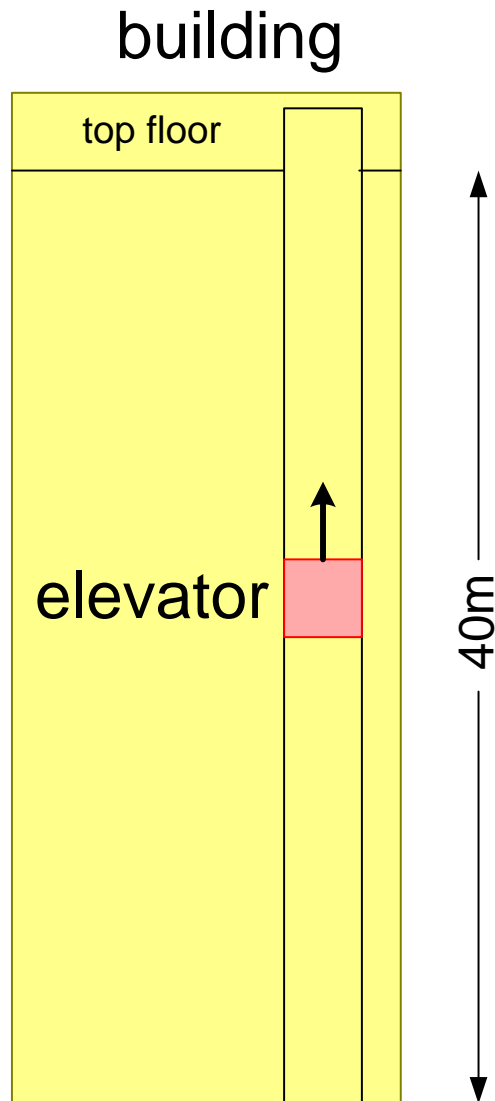
$t_{\text{top floor}}$ = time to reach top floor

v_{max} = maximum velocity

a_{max} = maximum acceleration

j_{max} = maximum jerk

Initial Estimates via Googling



Google "elevator" and "jerk":

$$t_{\text{top floor}} \approx 16 \text{ s}$$

$$v_{\text{max}} \approx 2.5 \text{ m/s}$$

12% of gravity;
weight goes up

$$a_{\text{max}} \approx 1.2 \text{ m/s}^2 \text{ (up)}$$

relates to motor design
and energy consumption

$$j_{\text{max}} \approx 2.5 \text{ m/s}^3 \text{ ——— relates to control design}$$

humans feel changes of forces
high jerk values are uncomfortable

numbers from: http://www.sensor123.com/vm_eva625.htm
CEP Instruments Pte Ltd Singapore

Exercise Time to Reach Top Floor Kinematic

input data

$$S_0 = 0\text{m} \quad S_t = 40\text{m}$$

$$v_{\max} = 2.5 \text{ m/s}$$

$$a_{\max} = 1.2 \text{ m/s}^2 \text{ (up)}$$

$$j_{\max} = 2.5 \text{ m/s}^3$$

elementary formulas

$$v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

exercises

$t_{\text{top floor}}$ is time needed to reach top floor without stopping

Make a model for $t_{\text{top floor}}$ and calculate its value

Make 0^e order model, based on constant velocity

Make 1^e order model, based on constant acceleration

What do you conclude from these models?

Models for Time to Reach Top Floor

input data

$$S_0 = 0\text{m} \quad S_{\text{top floor}} = 40\text{m}$$

$$v_{\text{max}} = 2.5 \text{ m/s}$$

$$a_{\text{max}} = 1.2 \text{ m/s}^2 \text{ (up)}$$

$$j_{\text{max}} = 2.5 \text{ m/s}^3$$

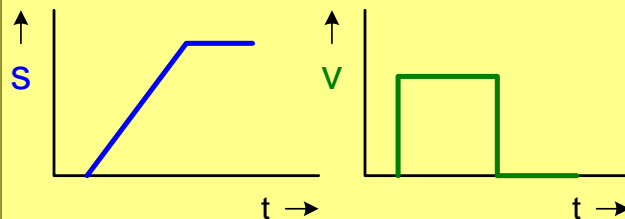
elementary formulas

$$v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

0th order model

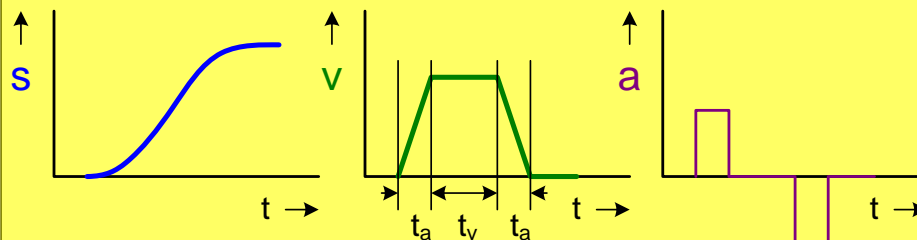


$$S_{\text{top floor}} = v_{\text{max}} * t_{\text{top floor}}$$

$$t_{\text{top floor}} = S_{\text{top floor}} / v_{\text{max}}$$

$$t_{\text{top floor}} = 40/2.5 = \mathbf{16\text{s}}$$

1st order model



$$t_a \approx 2.5/1.2 \approx 2\text{s}$$

$$S(t_a) \approx 0.5 * 1.2 * 2^2$$

$$S(t_a) \approx 2.4\text{m}$$

$$t_v \approx (40 - 2 * 2.4) / 2.5$$

$$t_v \approx 14\text{s}$$

$$t_{\text{top floor}} = t_a + t_v + t_a$$

$$S_{\text{linear}} = S_{\text{top floor}} - 2 * S(t_a)$$

$$t_{\text{top floor}} \approx 2 + 14 + 2$$

$$t_{\text{top floor}} \approx \mathbf{18\text{s}}$$

$$t_a = v_{\text{max}} / a_{\text{max}}$$

$$t_v = S_{\text{linear}} / v_{\text{max}}$$

$$S(t_a) = \frac{1}{2} * a_{\text{max}} * t_a^2$$

Conclusions

v_{\max} dominates traveling time

The model for the large height traveling time can be simplified into:

$$t_{\text{travel}} = S_{\text{travel}}/v_{\max} + (t_a + t_j)$$

Exercise Time to Travel One Floor

input data

$$S_0 = 0\text{m} \quad S_{\text{top floor}} = 40\text{m}$$

$$v_{\text{max}} = 2.5 \text{ m/s}$$

$$a_{\text{max}} = 1.2 \text{ m/s}^2 \text{ (up)}$$

$$j_{\text{max}} = 2.5 \text{ m/s}^3$$

elementary formulas

$$v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

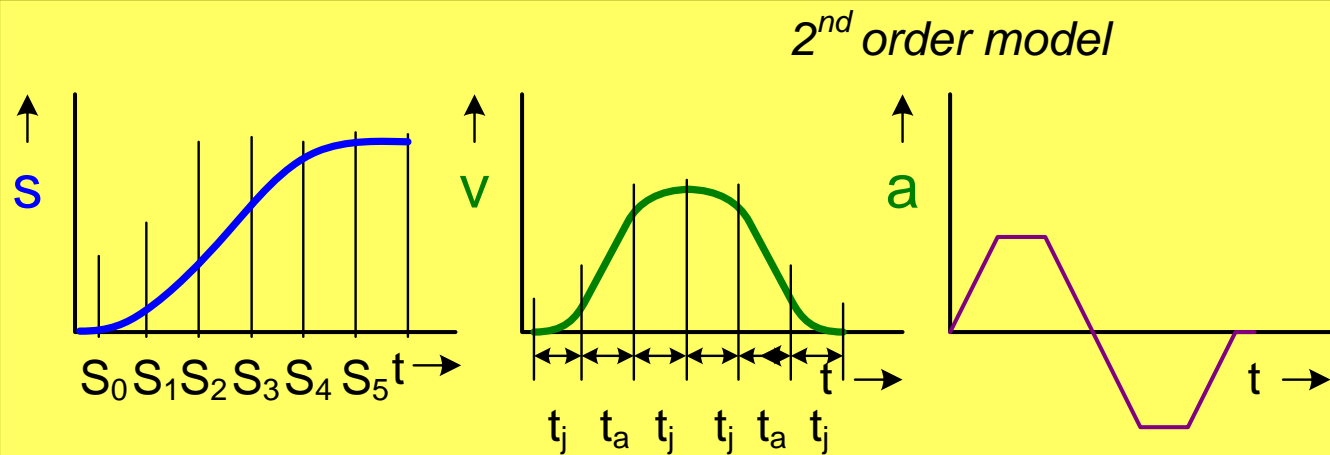
$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

exercise

Make a model for $t_{\text{one floor}}$ and calculate it

What do you conclude from this model?

2nd Order Model Moving One Floor



input data

$$S_0 = 0\text{m}$$

$$S_{\text{one floor}} = 3\text{m}$$

$$v_{\text{max}} = 2.5 \text{ m/s}$$

$$a_{\text{max}} = 1.2 \text{ m/s}^2 \text{ (up)}$$

$$j_{\text{max}} = 2.5 \text{ m/s}^3$$

$$t_{\text{one floor}} = 2 t_a + 4 t_j$$

$$t_j = a_{\text{max}} / j_{\text{max}}$$

$$S_1 = 1/6 * j_{\text{max}} t_j^3$$

$$v_1 = 0.5 j_{\text{max}} t_j^2$$

$$S_2 = S_1 + v_1 t_a + 0.5 a_{\text{max}} t_a^2$$

$$v_2 = v_1 + a_{\text{max}} t_a$$

$$S_3 = S_2 + v_2 t_j + 0.5 a_{\text{max}} t_j^2 - 1/6 j_{\text{max}} t_j^3$$

$$S_3 = 0.5 S_t$$

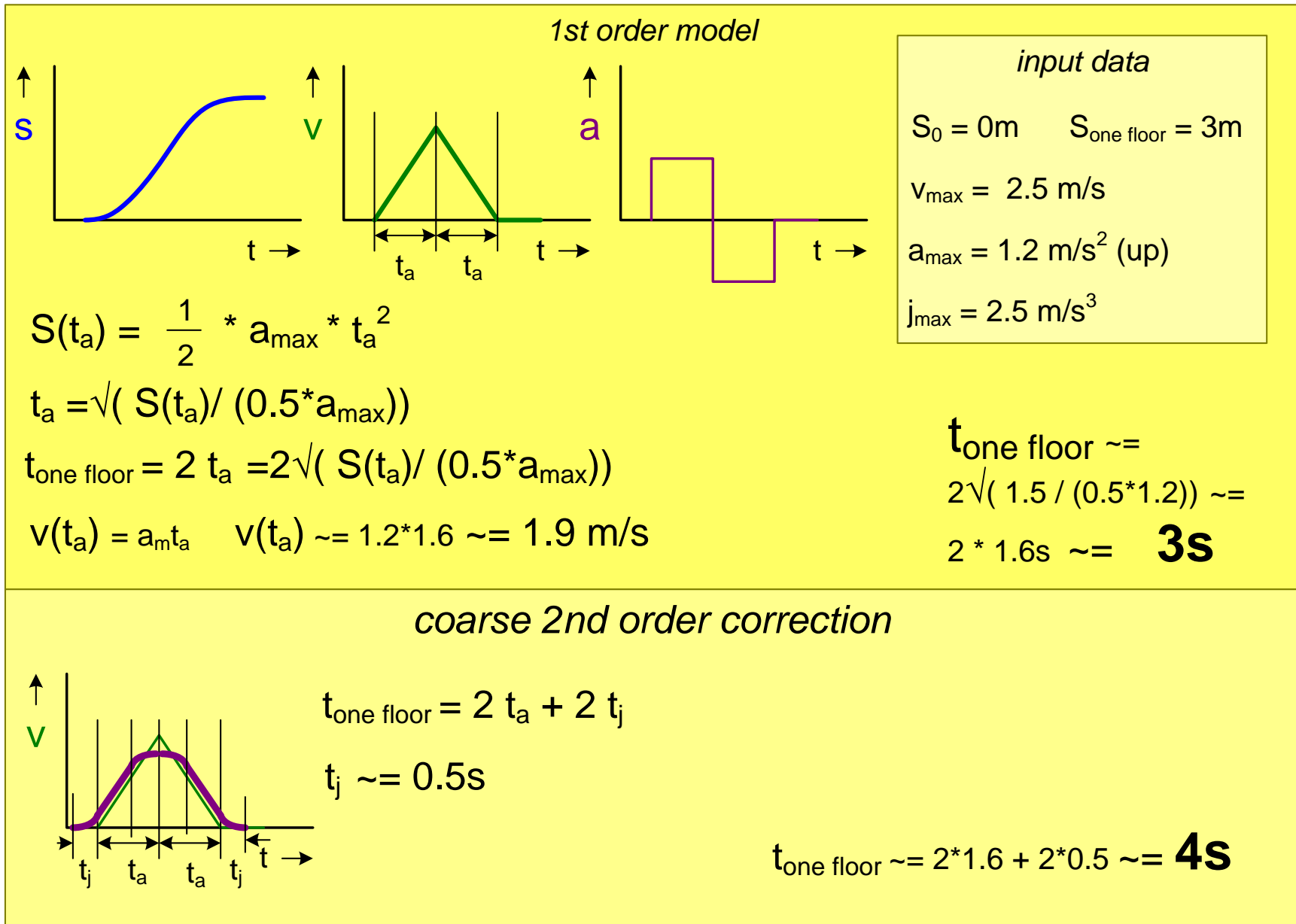
$$t_j \approx 1.2/2.5 \approx 0.5\text{s}$$

$$S_1 \approx 1/6 * 2.5 * 0.5^3 \approx 0.05\text{m}$$

$$v_1 \approx 0.5 * 2.5 * 0.5^2 \approx 0.3\text{m/s}$$

et cetera

1st Order Model Moving One Floor



Conclusions

a_{\max} dominates travel time

The model for small height traveling time can be simplified into:

$$t_{\text{travel}} = 2 \sqrt{(S_{\text{travel}}/0.5 a_{\max})} + t_j$$

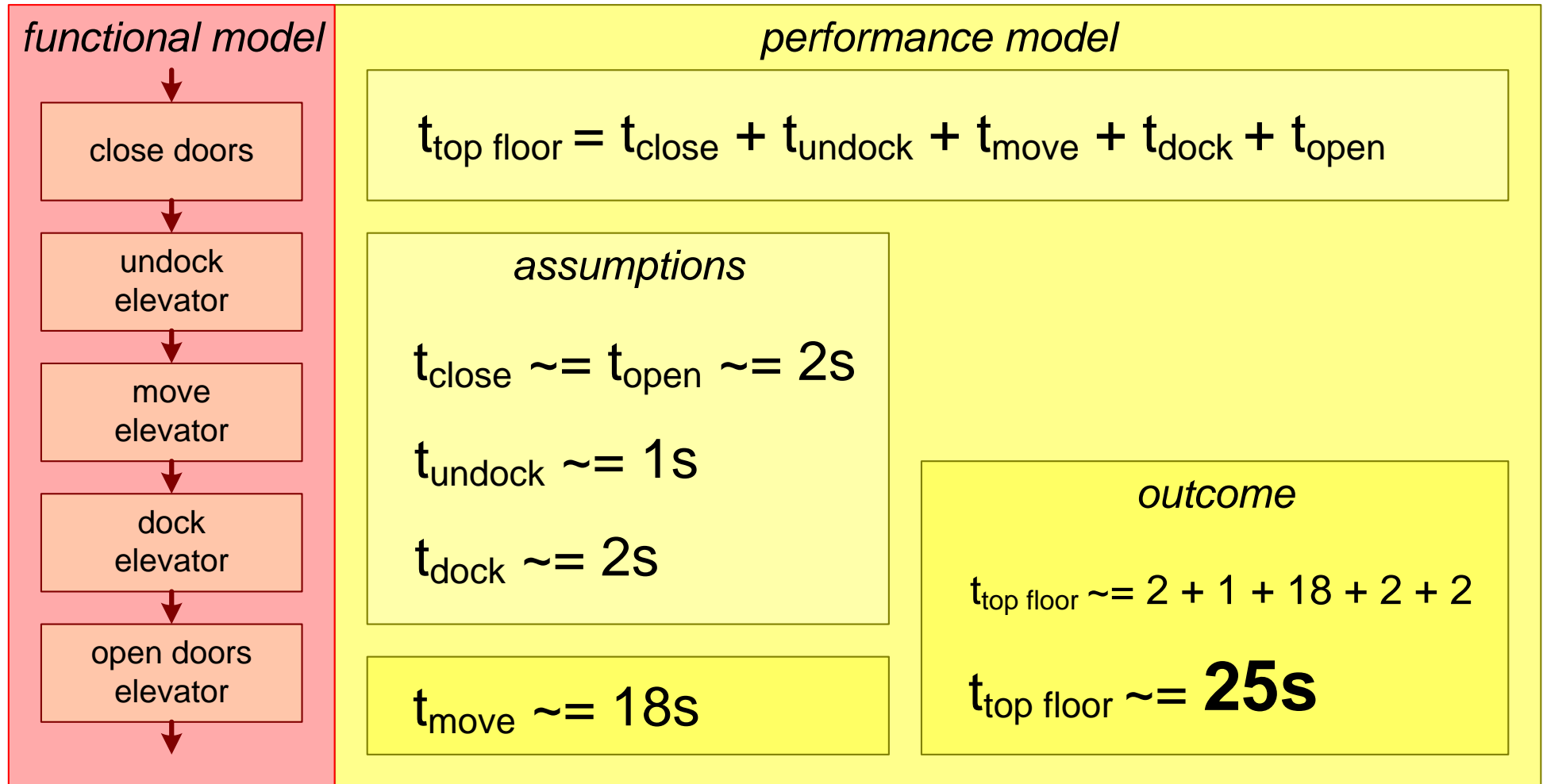
exercise

Make a model for $t_{\text{top floor}}$

Take door opening and docking into account

What do you conclude from this model?

Elevator Performance Model



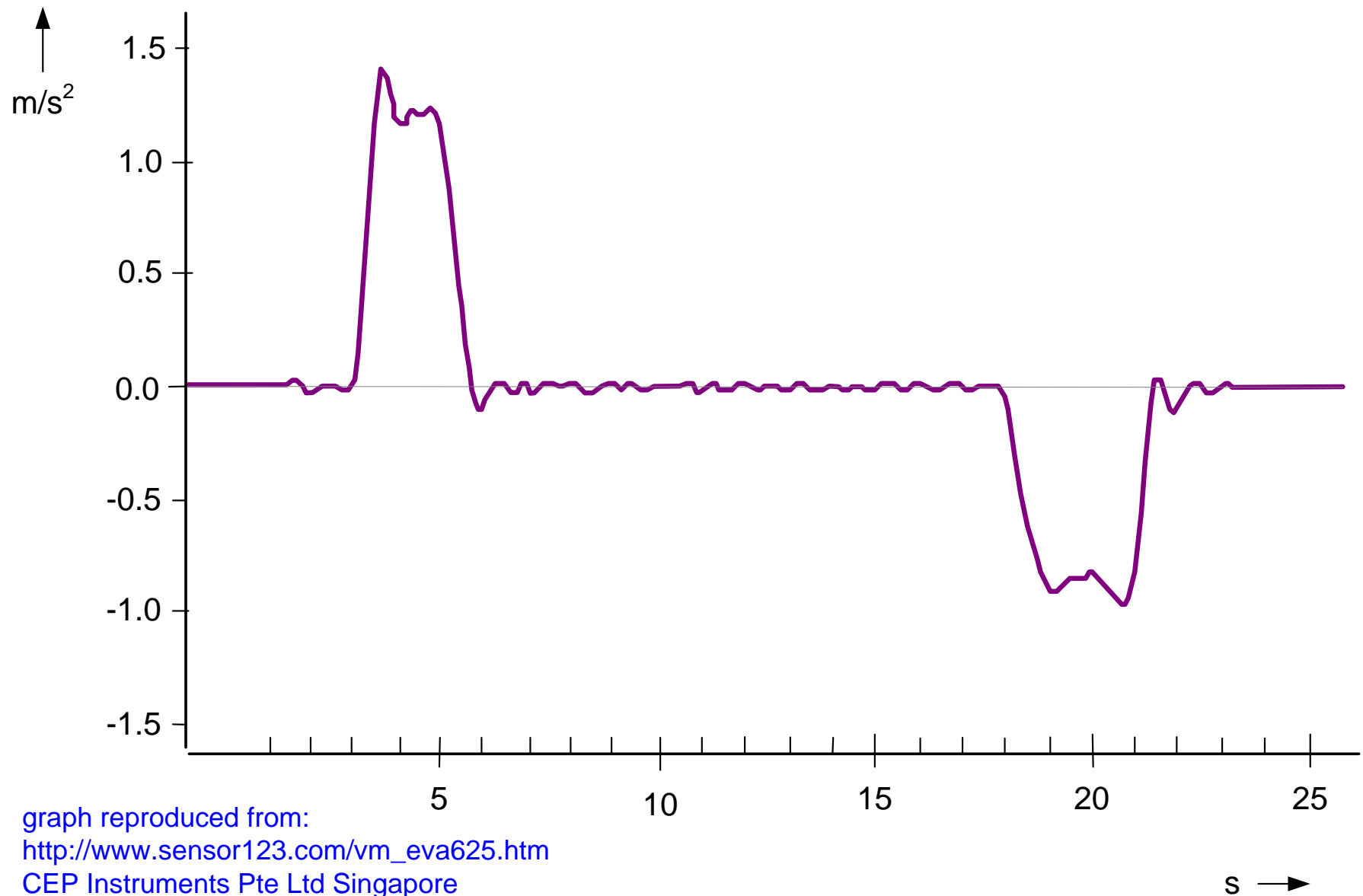
Conclusions

The time to move is dominating the traveling time.

Docking and door handling is significant part of the traveling time.

$$t_{\text{top floor}} = t_{\text{travel}} + t_{\text{elevator overhead}}$$

Measured Elevator Acceleration



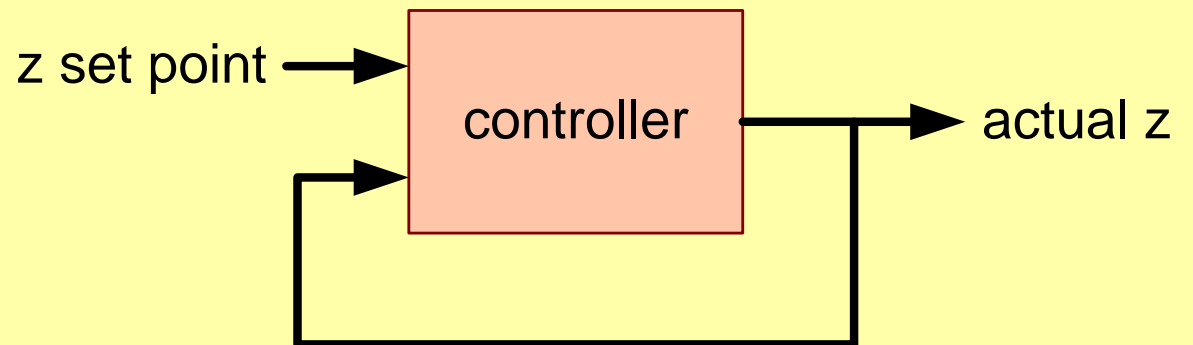
What did we ignore or forget?

acceleration: up \leftrightarrow down 1.2 m/s^2 vs 1.0 m/s^2

slack, elasticity, damping et cetera of cables, motors....

controller impact

.....



Exercise Time to Travel One Floor

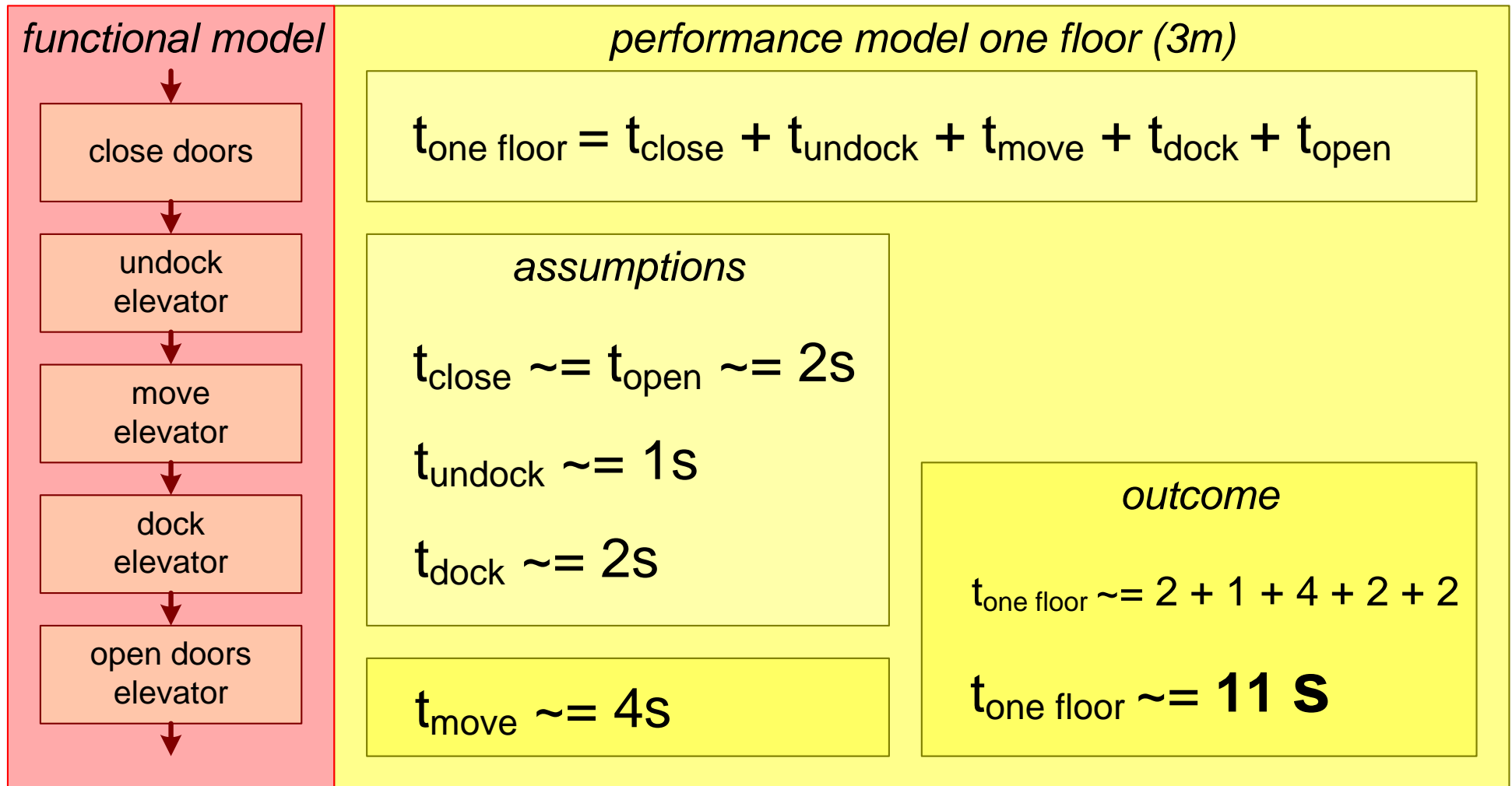
exercise

Make a model for $t_{\text{one floor}}$

Take door opening and docking into account

What do you conclude from this model?

Elevator Performance Model



Conclusions

Overhead of docking and opening and closing doors is dominating traveling time.

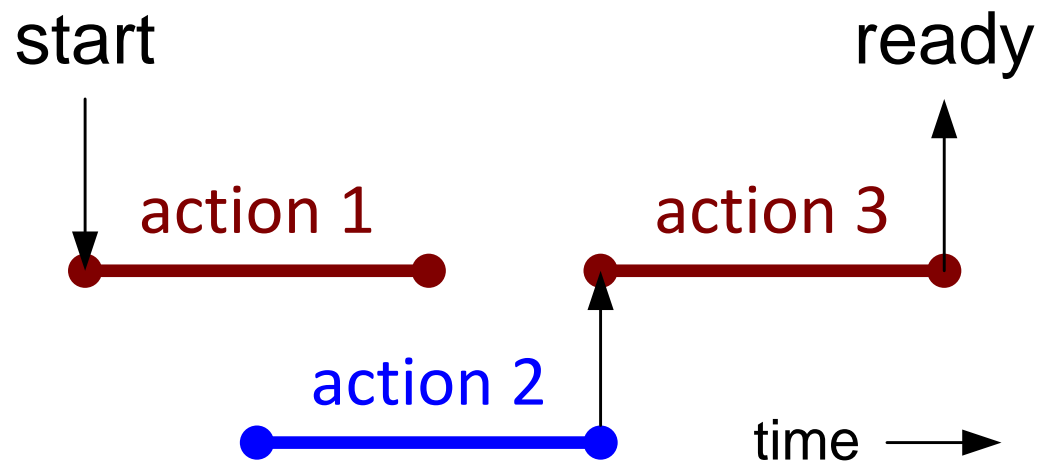
Fast docking and fast door handling has significant impact on traveling time.

$$t_{\text{one floor}} = t_{\text{travel}} + t_{\text{elevator overhead}}$$

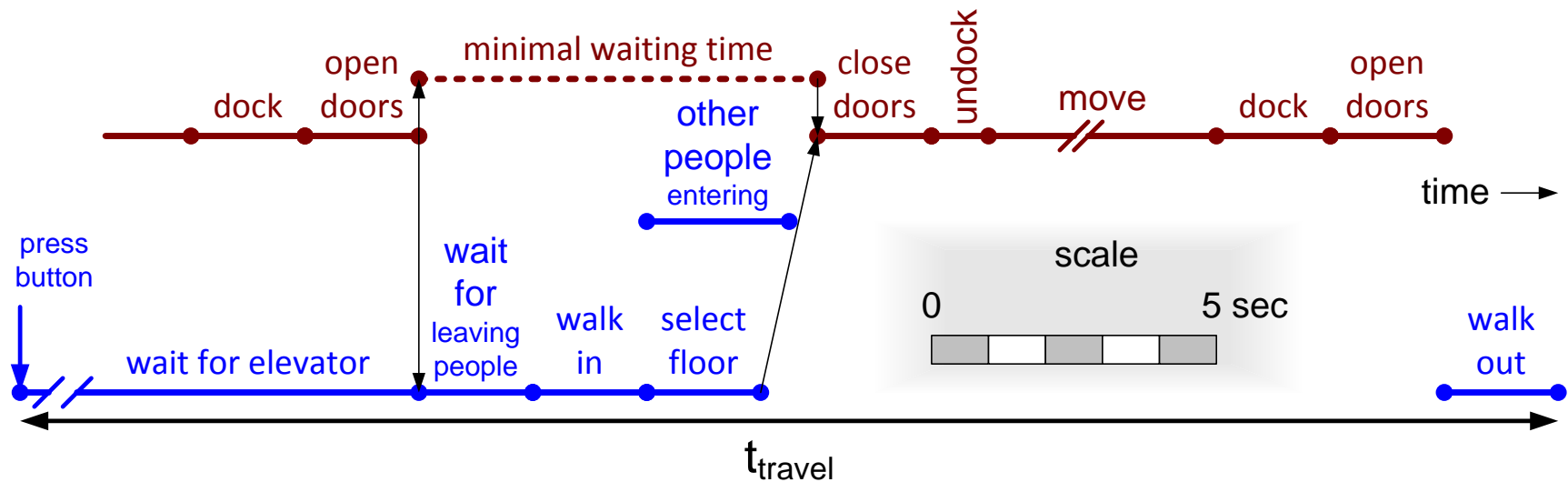
Exercise Time Line

Exercise

Make a time line of people using the elevator.
Estimate the time needed to travel to the top floor.
Estimate the time needed to travel one floor.
What do you conclude?



Time Line; Humans Using the Elevator



assumptions human dependent data

$t_{\text{wait for elevator}} = [0..2 \text{ minutes}]$ depends heavily on use

$t_{\text{wait for leaving people}} = [0..20 \text{ seconds}]$ idem

$t_{\text{walk in}} \approx t_{\text{walk out}} \approx 2 \text{ s}$

$t_{\text{select floor}} \approx 2 \text{ s}$

assumptions additional elevator data

$t_{\text{minimal waiting time}} \approx 8 \text{ s}$

$t_{\text{travel top floor}} \approx 25 \text{ s}$

$t_{\text{travel one floor}} \approx 11 \text{ s}$

outcome

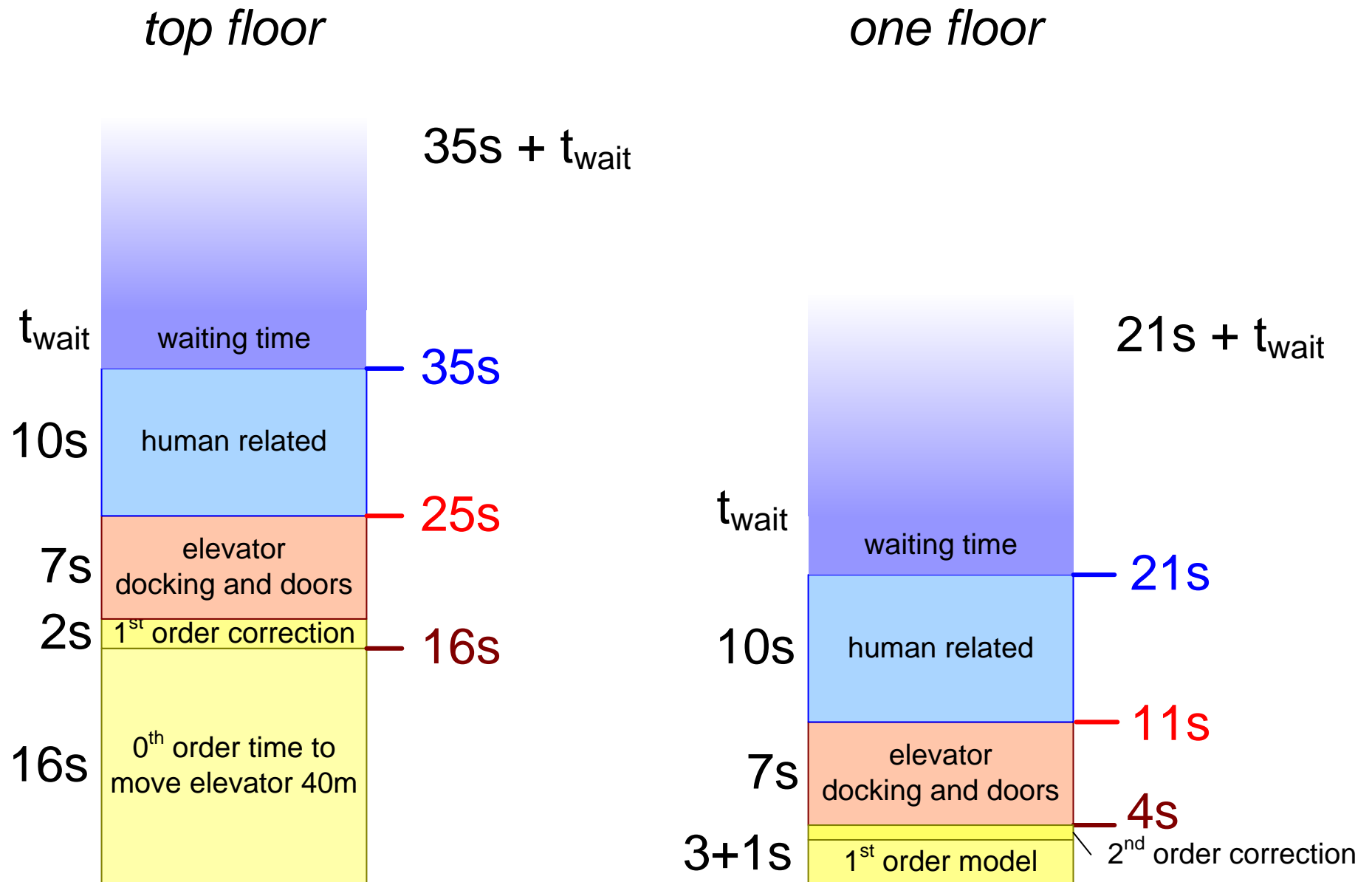
$$t_{\text{one floor}} = t_{\text{minimal waiting time}} + t_{\text{walk out}} + t_{\text{travel one floor}} + t_{\text{wait}}$$

$$t_{\text{top floor}} = t_{\text{minimal waiting time}} + t_{\text{walk out}} + t_{\text{travel top floor}} + t_{\text{wait}}$$

$$t_{\text{one floor}} \approx 8 + 2 + 11 + t_{\text{wait}} \\ \approx \mathbf{21 \text{ s}} + t_{\text{wait}}$$

$$t_{\text{top floor}} \approx 8 + 2 + 25 + t_{\text{wait}} \\ \approx \mathbf{35 \text{ s}} + t_{\text{wait}}$$

Overview of Results for One Elevator



Conclusions

The human related activities have significant impact on the end-to-end time.

The waiting times have significant impact on the end-to-end time and may vary quite a lot.

$$t_{\text{end-to-end}} = t_{\text{human activities}} + t_{\text{wait}} + t_{\text{elevator travel}}$$

Exercise

Estimate the energy consumption and the average and peak power needed to travel to the top floor.

What do you conclude?

Energy and Power Model

input data

$S_0 = 0\text{m}$ $S_t = 40\text{m}$
 $v_{\text{max}} = 2.5\text{ m/s}$ $m_{\text{elevator}} = 1000\text{ Kg}$ (incl counter weight)
 $a_{\text{max}} = 1.2\text{ m/s}^2$ (up) $m_{\text{passenger}} = 100\text{ Kg}$
 $j_{\text{max}} = 2.5\text{ m/s}^3$ 1 passenger going up
 $g = 10\text{ m/s}^2$

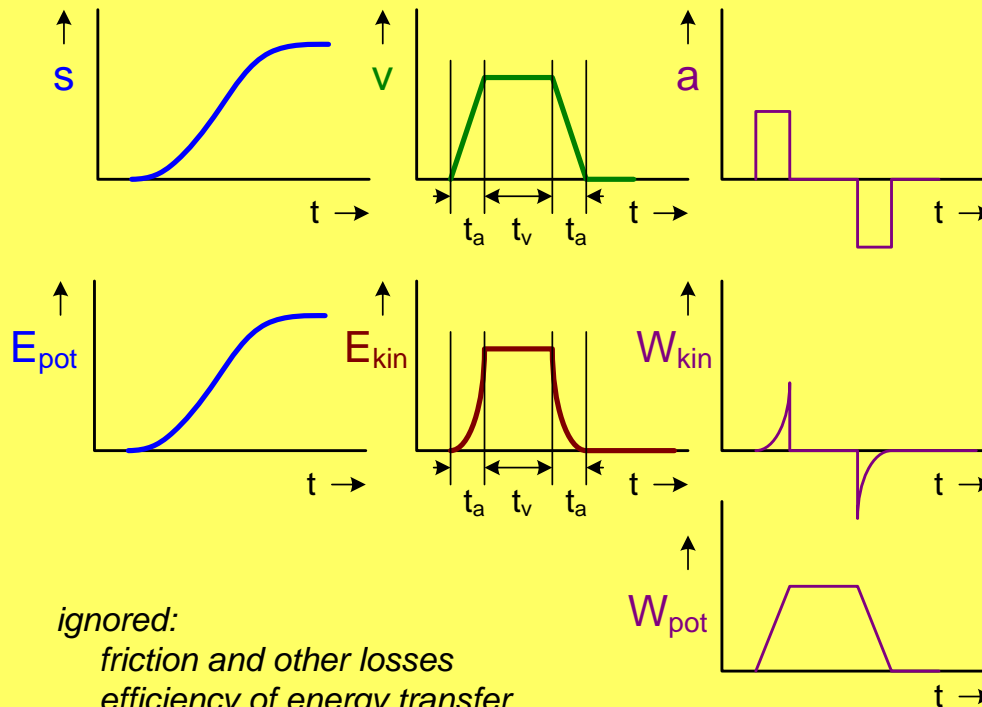
elementary formulas

$$E_{\text{kin}} = 1/2 m v^2$$

$$E_{\text{pot}} = mgh$$

$$W = \frac{dE}{dt}$$

1st order model



$$E_{\text{kin max}} = 1/2 m v_{\text{max}}^2$$

$$\sim 0.5 * 1100 * 2.5^2$$

$$\sim \mathbf{3.4\text{ kJ}}$$

$$W_{\text{kin max}} = m v_{\text{max}} a_{\text{max}}$$

$$\sim 1100 * 2.5 * 1.2$$

$$\sim \mathbf{3.3\text{ kW}}$$

$$E_{\text{pot}} = mgh$$

$$\sim 100 * 10 * 40$$

$$\sim \mathbf{40\text{ kJ}}$$

$$W_{\text{pot max}} \sim E_{\text{pot}}/t_v$$

$$\sim 40/16$$

$$\sim \mathbf{2.5\text{ kW}}$$

Energy and Power Conclusions

Conclusions

E_{pot} dominates energy balance

W_{pot} is dominated by v_{max}

W_{kin} causes peaks in power consumption and absorption

W_{kin} is dominated by v_{max} and a_{max}

$$E_{\text{kin max}} = 1/2 m v_{\text{max}}^2$$
$$\approx 0.5 * 1100 * 2.5^2$$
$$\approx \mathbf{3.4 \text{ kJ}}$$

$$W_{\text{kin max}} = m v_{\text{max}} a_{\text{max}}$$
$$\approx 1100 * 2.5 * 1.2$$
$$\approx \mathbf{3.3 \text{ kW}}$$

$$E_{\text{pot}} = mgh$$
$$\approx 100 * 10 * 40$$
$$\approx \mathbf{40 \text{ kJ}}$$

$$W_{\text{pot max}} \approx E_{\text{pot}}/t_v$$
$$\approx 40/16$$
$$\approx \mathbf{2.5 \text{ kW}}$$

Exercise

What other qualities and design considerations relate to the kinematic models?

Conclusions Qualities and Design Considerations

Examples of other qualities and design considerations

safety

v_{\max}

acoustic noise

$v_{\max}, a_{\max}, j_{\max}$

mechanical vibrations

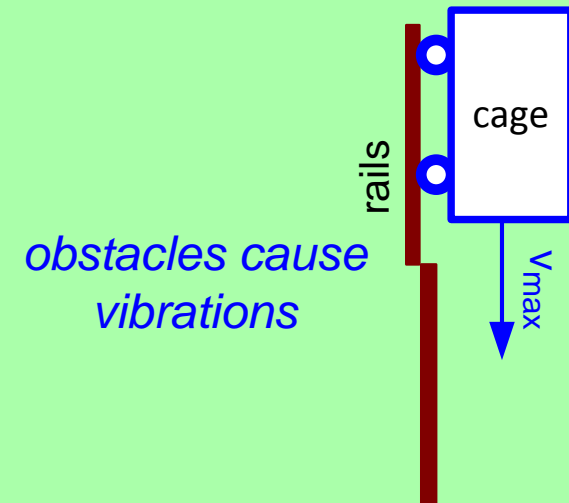
$v_{\max}, a_{\max}, j_{\max}$

air flow

?

operating life, maintenance duty cycle, ?

...



applicability in other domains

kinematic modeling can be applied in a wide range of domains:

transportation systems (trains, busses, cars, containers, ...)

wafer stepper stages

health care equipment patient handling

material handling (printers, inserters, ...)

MRI scanners gradient generation

...

Exercise

Assume that a group of people enters the elevator at the ground floor. On every floor one person leaves the elevator.

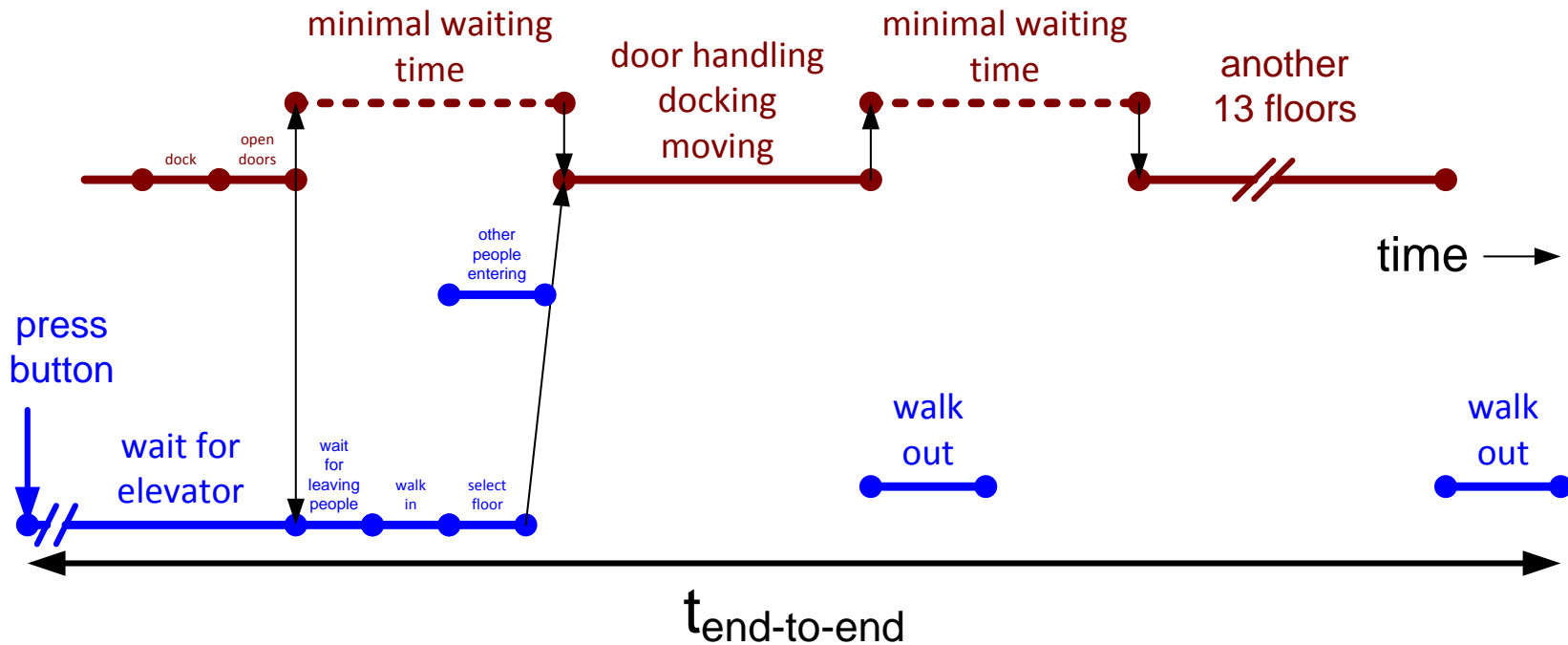
What is the end-to-end time for someone traveling to the top floor?

What is the desired end-to-end time?

What are potential solutions to achieve this?

What are the main parameters of the design space?

Multiple Users Model



<i>elevator data</i>
$t_{\min \text{ wait}} \approx 8\text{s}$
$t_{\text{one floor}} \approx 11\text{s}$
$t_{\text{walk out}} \approx 2\text{s}$
$n_{\text{floors}} = 40 \text{ div } 3 + 1 = 14$
$n_{\text{stops}} = n_{\text{floors}} - 1 = 13$

<i>outcome</i>
$t_{\text{end-to-end}} = n_{\text{stops}} (t_{\min \text{ wait}} + t_{\text{one floor}}) + t_{\text{walk out}} + t_{\text{wait}}$ $\approx 13 * (8 + 11) + 2 + t_{\text{wait}}$ $\approx \mathbf{249 \text{ s}} + t_{\text{wait}}$

$t_{\text{non-stop}} \approx \mathbf{35 \text{ s}} + t_{\text{wait}}$

Multiple Users Desired Performance

Considerations

desired time to travel to top floor $\sim < 1$ minute

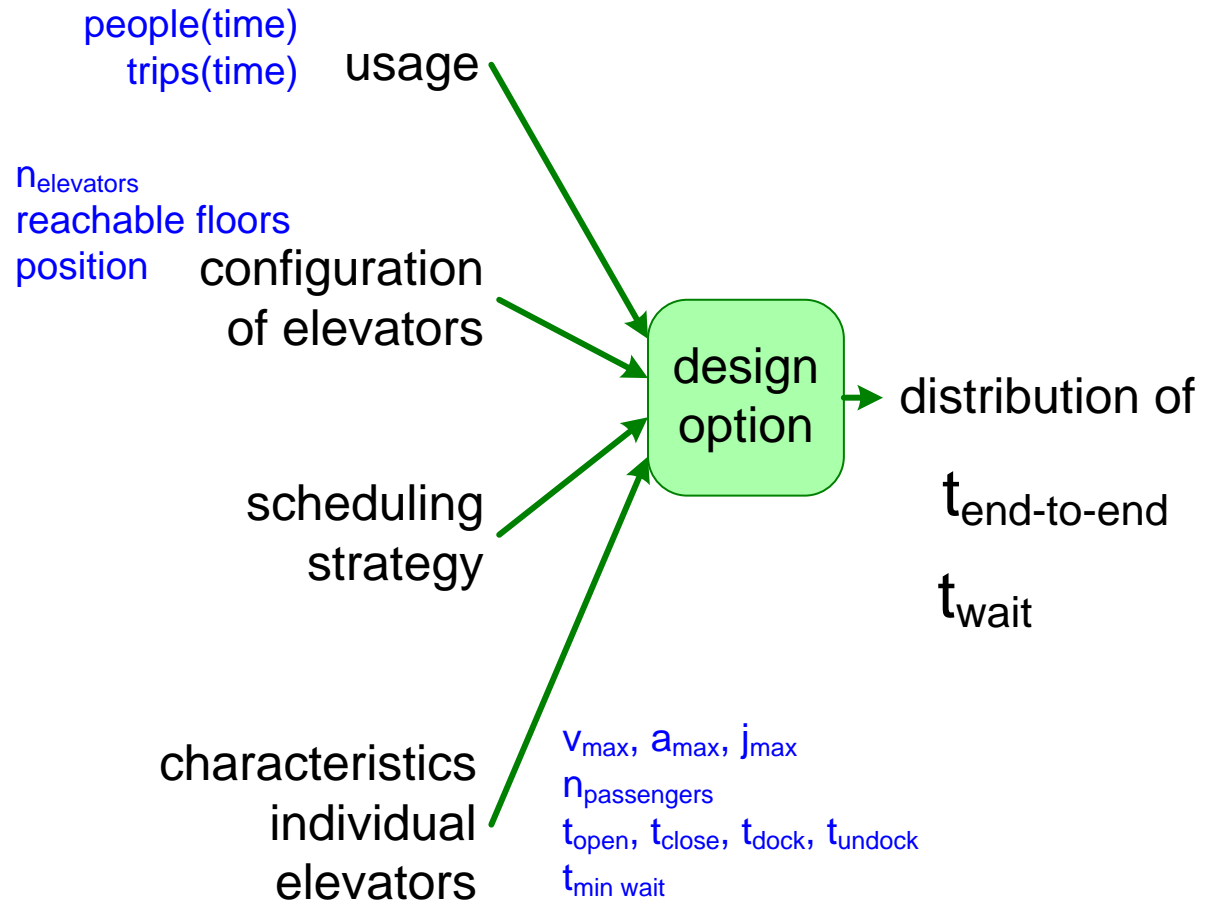
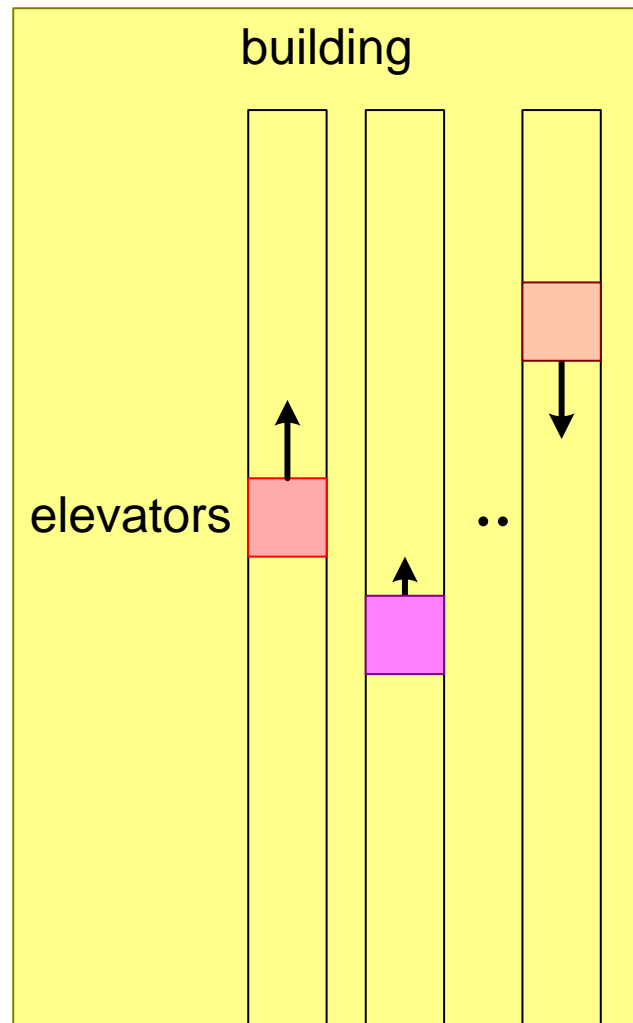
note that $t_{\text{wait next}} = t_{\text{travel up}} + t_{\text{travel down}}$

if someone just misses the elevator then the waiting time is

$t_{\text{end-to-end}} \sim = \overset{\text{missed}}{\underset{\text{trip}}{249}} + \overset{\text{return}}{\underset{\text{down}}{35}} + \overset{\text{trip}}{\underset{\text{up}}{249}} = 533\text{s} \sim = 9 \text{ minutes!}$

desired waiting time $\sim < 1$ minute

Design of Elevators System



Design of a system with multiple elevator requires a different kind of models: oriented towards logistics

Exceptional Cases

non-functioning elevator

maintenance, cleaning of elevator

elevator used by people moving household

rush hour

special events (e.g. party, new years eve)

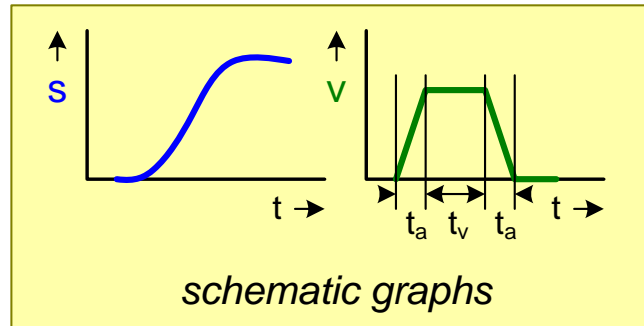
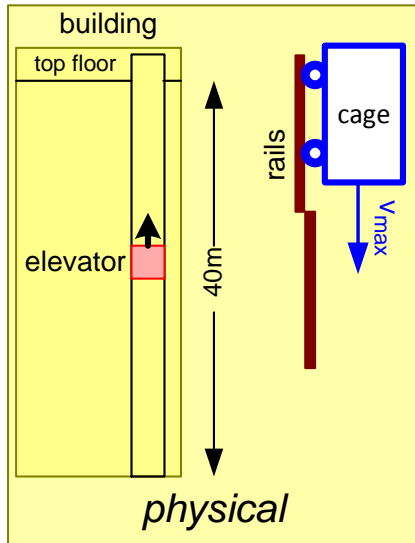
special floors (e.g. restaurant)

many elderly or handicapped people

playing children

Make a list of all *visualizations* and *representations* that we used during the exercises

Summary of Visualizations and Representations



$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

$$t_{\text{top floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}}$$

mathematical formulas

